Homework on Interpreting the Derivative[i]

1. Consider the *Garfield* comic from July 24, 1992.

Let $W(t)$ be Garfield's weight as a function of time.

a. Rephrase Jon's last comment in terms of calculus by filling in each blank with one or more items from among the following expressions:

$W < 0$ $\quad W > 0$ $\quad W' < 0$ $\quad W' > 0$ $\quad W'' < 0$ $\quad W'' > 0$

"With Garfield, the goal of a diet isn't _______________, it's ___________________."

b. Sketch a graph of Garfield's weight over time.

2. Susie and Rebecca decide to get in shape by running. Let $S(t)$ and $R(t)$ be the distances that Susie and Rebecca can run on day t, respectively. Interpret the following symbolic statements in words.

a. $S(1) = \frac{1}{2}$ mile

b. $R(1) = 2S(1)$

c. $S'(\text{t}) = \frac{1}{8}$ mile per day for $1 \le t \le 10$

d. $R'(\text{t}) = \frac{1}{10}$ mile per day for $1 \le t \le 10$

e. At the end of 10 days, who is running the farthest on her daily run? Explain your reasoning.

3. Let $H(t)$ be the number of dollars spent on health care in the US in the year *t*. (Assume *t* = 0 is the year 2006.)

   "Rise in health spending slows; Overall growth rate declines for first time in seven years"- Atlanta Journal Constitution, January 18, 2006.

Preparatory workout: Here are some things to consider before answering the question.

$H(0)$ is . . .	Positive?	Negative?	Zero?	
When $t = 0$, $H(t)$ is . . .	Increasing?	Decreasing?	Zero?	
$H'(0)$ is . . .	Positive?	Negative?	Zero?	
When $t = 0$, $H'(t)$ is . . .	Increasing?	Decreasing?	Zero?	
When $-7 \le t < 0$, $H(t)$ is . . .	Positive?	Negative?	Increasing?	Decreasing?
When $-7 \le t < 0$, $H'(t)$ is . . .	Positive?	Negative?	Increasing?	Decreasing?

What does this information tell you about the second derivative of H? That is, about H'' *?*

Now Translate the headline into the language of calculus using first and second derivatives.

Choose *either* problem 4 or problem 5.

4. Suppose that A(p) gives the number of pounds of apples sold as a function of the price (in dollars) per pound.

   a. What are the units of $\frac{dA}{dp}$?

   b. Do you expect $\frac{dA}{dp}$ to be positive or negative? Why?

   c. Interpret the statement $A'(.88) = -50$ in words.

   d. Suppose that you sold 500 pounds of apples at $.88 per pound. Use $A'(.88) = -50$ to estimate how many apples you will sell if you charge $1 per pound this week.

5. During a chemical reaction, the concentration of a certain chemical at time t (in minutes) in a beaker is measured in moles per liter. Suppose that the rate of change of this concentration is given by $C'(t) = \frac{10}{(t+5)^2}$ where time is measured in minutes.

   a. What is the rate at which C is changing at $t = 2$ minutes? (Be sure to use correct units.)

   b. Suppose that $C(2) \approx .57$ moles per liter. Use your answer to part a. to *estimate* the concentration of the chemical at 2.3 minutes.

   c. What assumption did you make during your computation in part b?

   d. Is your estimate larger or smaller than the actual concentration?
   Hint: remember that concentrations are always positive. Now get Maple to help you plot a graph of $C'(t) = \frac{10}{(t+5)^2}$. What does this tell you about the *shape* of the graph of C at $t = 2$?

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[i] These problems are adapted from various sources. Some were adapted by a project created by Prof. Judy Holdener. Others were adapted from problems in *Calculus: Graphical, Numerical, and Analytic Points of View* by A. Ostebee and P. Zorn and in *Calculus: the Language of Change* by David Cohen and James Henle. (Some were adapted by Prof. Bob Milnikel, others by Prof. Carol Schumacher)